Using Diversity of Message Passing Algorithms for Error Correcting Codes

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New Message Passing Algorithms Worshop, Orsay, Oct. 2009

Outline

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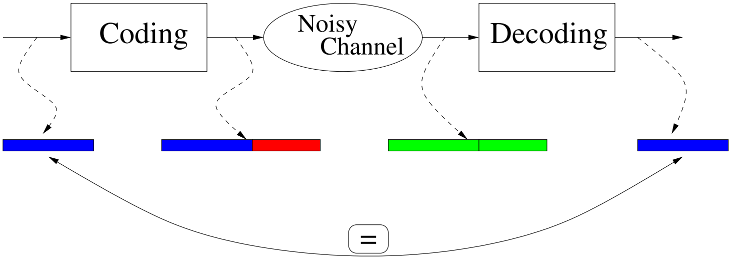
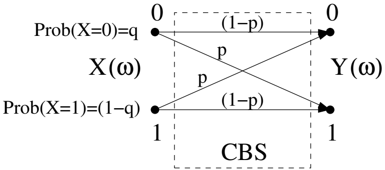
Coding = Increase Message Length by Adding Redundancy



Rate of the Code : $R = \dfrac{\text{info.}}{\text{info. + red.}}$

Considered Transmission Channel

Binary Symmetric Channel



Codeword	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Noisy Codeword	0	0	0	**1**	**1**	0	0	**1**	0	0	0	0	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0	**1**	0

Definitions and Quantities

Code

$$\mathcal{C}_H = \left\{ \underline{c} \in GF(2)^{\times N} \;\middle|\; H.\underline{c} \overset{GF(2)}{=} 0 \right\}$$

Code

$$\mathcal{C}_G = \{ \underline{c} = G.\underline{u}, \forall \underline{u} \in GF(2)^{\times K} \}$$

with $H(M \times N)$, parity check matrix,
with $G(N \times K)$, generator matrix.

Size	of a codeword	N	$R = K/N$
	of information	K	(if H is full rank)
	of the redundancy	M	

Link: Optimal Decoding $\leftrightarrow$ Hamming Distance

- Maximum Likelihood Decoding:

$$\hat{c}_{MLD} = arg \max_{c \in \mathcal{C}} p(y|c)$$

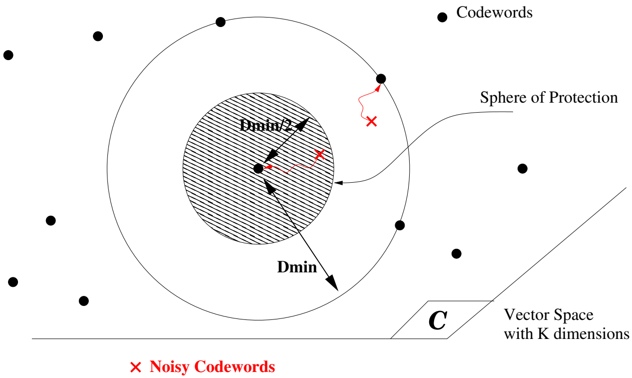
- Minimum Hamming Distance:

$$d_H(c_1, c_2) = \sum_{n=1}^{N} \|c_1(n) - c_2(n)\|$$

$$d_{min} = \min_{c \in \mathcal{C}} d_H(c, 0)$$

Minimum Hamming Weight and Error Correction

MLD ⇔ minimize Hamming Distance (BSC)



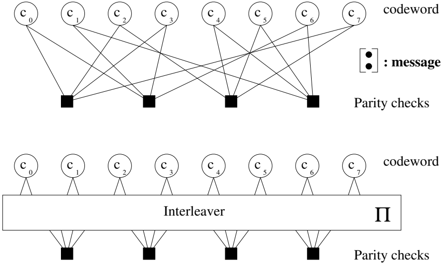
PB: Maximum Likelihood Decoding is NP-Complete

Approximate Solution: Belief Propagation on Sparse codes

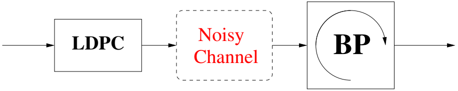
Tanner Graph is a Bi-partite graph with Adjacency Matrix H

LDPC: **L**ow **D**ensity **P**arity **C**heck Codes

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

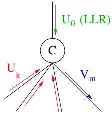


Belief Propagation Algorithm in the Log-Domain



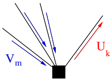
$$u_k = \log \frac{u_k[0]}{u_k[1]}$$

message update through 2 types of nodes

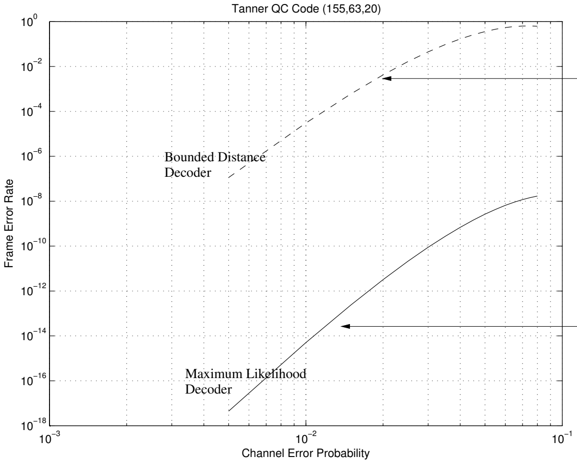


$$v_m = u_0 + \sum_{k=1,k\neq m}^{i} u_k$$

$$\tanh \frac{u_k}{2} = \prod_{m=1;m\neq k}^{j} \tanh \frac{v_m}{2}$$



Current Limits of Iterative Decoders



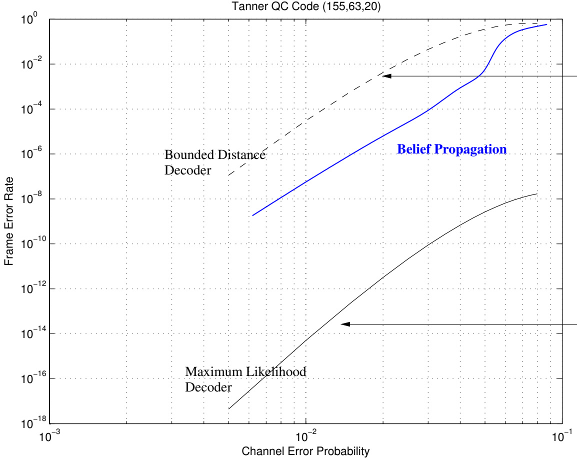
Proof of polynomial complexity (algebraic approach)

Easy to beat with linear complexity message passing

Proof of Exponential complexity Problem

How close can we get with message passing ?

Current Limits of Iterative Decoders



Proof of polynomial complexity (algebraic approach)

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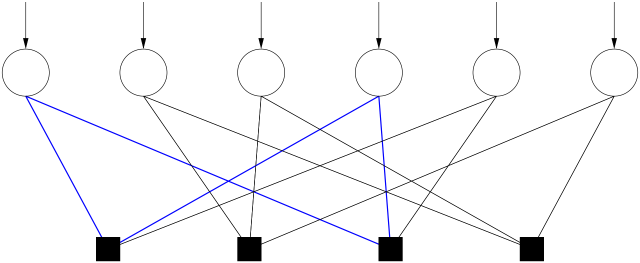
How close can we get with message passing ?

Current Limits of Iterative Decoders

Sub-optimality comes from presence of loops in the Tanner Graph



Outline

Lets go back to the Decoding Problem of non-Sparse Codes

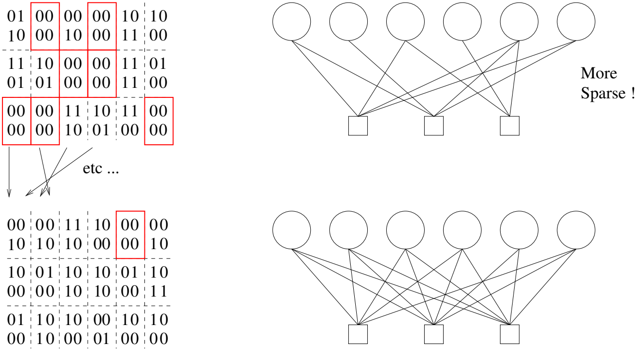
Problem statement

- Cyclic Dense codes have very good minimum distance : BCH codes (Bose, Chaudry, Hockenheim),
- They have the potential to correct a large proportion of errors, with a *efficient decoder*,
- PB : on the BSC, no better *practical* algorithm than Bounded Distance Decoder is known.
- Message Passing Decoders are even worse because the code is not sparse (huge number of minimum cycles).

Proposed Solution

- Transform the Tanner graph into a Non-Binary Factor graph by clustering rows and columns of H,
- Use many instances of Belief Propagation on the transformed Tanner Graph.

Good and Bad Tanner Graphs from the same *H*:



Good and Bad ?

A small example (2)
Diversity of Decoding behaviors

test case: 10000 noise vectors at Channel Error Probability $\alpha = 0.1$

- **5019** cases where both decoders converge to the right codeword,
- **384** cases where both decoders fail to converge after 50 iterations,
- **589** cases where both decoders converge to a wrong codeword,
- **1213** cases where one decoder fails to converge, and the other one converges to the right codeword,
- **1041** cases where one decoder fails to converge, and the other one converges to a wrong codeword,
- **1754** cases where one decoder converges to a wrong codeword, and the other one converges to the right codeword.

Good cases Bad cases

Capitalizing on the Decoding Diversity (1)

Diversity Sets

- **Definition of Diversity Set:** set of d distinct non-binary Tanner graphs of the same code:

$$\left\{\mathcal{G}_{p_1}^{(1)}, \ldots, \mathcal{G}_{p_i}^{(i)}, \ldots, \mathcal{G}_{p_d}^{(d)}\right\}$$

with

1. $H_b^{(i)} = \mathcal{P}^{(i)}\left(H_b\right) = A^{(i)}.H_b.\Pi^{(i)}$ is called pre-processing,
2. $\mathcal{G}_{p_i}^{(i)}$ is the Tanner graph obtained by clustering $H_b^{(i)}$ with an order p_i.

- **Diversity Sets examples:**
  - *Clustering diversity only*:
    $(A^{(i)} = Id_{M_b}, \Pi^{(i)} = Id_{N_b})_i \rightarrow \left\{\mathcal{G}_1^{(1)}, \dots, \mathcal{G}_9^{(9)}\right\}$,
  - *Constant clustering order and common code basis*:
    $(A^{(i)} = \Pi_1^{(i)}, \Pi^{(i)} = \Pi_2^{(i)})_i \rightarrow \left\{\mathcal{G}_p^{(1)}, \dots, \mathcal{G}_p^{(d)}\right\}$ have same number of nodes, and the same binary basis.
  - *Diversity set with binary Tanner graphs*:
    $p = 1$ (no clustering) $(A^{(i)}, \Pi^{(i)} = Id_{N_b}) \rightarrow$ the diversity set $\left\{\mathcal{G}_1^{(1)}, \dots, \mathcal{G}_1^{(d)}\right\}$ reduces to [Hehn, ISIT'2007].

Capitalizing on the Decoding Diversity (3)

Merging Strategies

- **Serial merging:** Use the decoders sequentially, *i.e.* switch to another decoder only when the current decoder failed to converge to a codeword,
  $$\Rightarrow \mathcal{O}(Serial) = (1+\varepsilon) \times \mathcal{O}(p.2^p.N.N_{it})$$
- **Parallel merging:** Run all *d* decoders in parallel and choose appropriately an estimated codeword (majority voting, maximum likelihood, etc)
  $$\Rightarrow \mathcal{O}(Parallel) = d \times \mathcal{O}(p.2^p.N.N_{it})$$
- **Other merging strategies:** $\mathcal{O}(Serial) \leq Comp \leq \mathcal{O}(Parallel)$,
- **Lower bound:** Is there at least 1 graph among the *d* candidates such that the decoder converges to the right codeword ? *(we do not address the PB of finding the good one among the d)*

NO information about Error Location is available

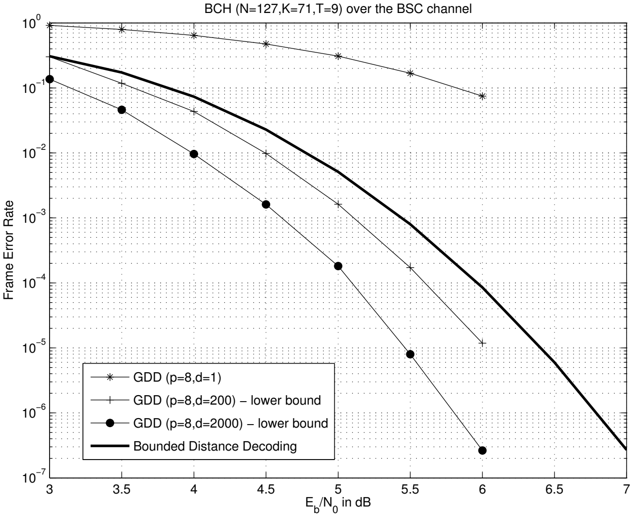
$\Rightarrow$ rely only on *random* preprocessing with sparsity constraint.

- Constant clustering order in the diversity sets p,
- Considered Preprocessing
  1. Build H_b from low weight codewords of the BCH dual code
     $\Rightarrow H_b$ is as sparse as possible,
  2. consider random row and column permutations:

$$H_b^{(i)} = \Pi_1^{(i)}.H_b.\Pi_2^{(i)}$$

BCH codes on the BSC Channel

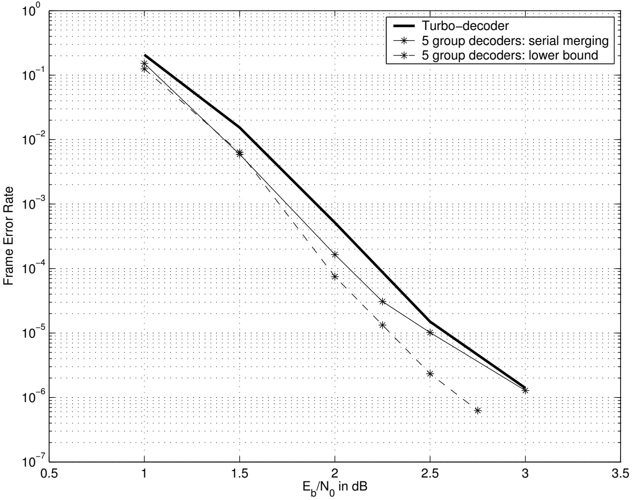
BCH($N = 127, K = 71, D_{min} = 19$)



Turbo-Codes from the DVB-RCS standard

Simulation results.

Diversity decoder performance for a $(R = 0.5, N = 848)$ duobinary TC



Outline

Perspectives of Research

1. **Non-Binary Belief Propagation Diversity**
   - Concept of Decoder Diversity is promising, but need of better understanding of non-binary Belief Propagation dynamics on dense graphs,
   - Clever choice of the diversity set of Tanner graphs ?
   - Adapt iteratively the NB factor graph.